Dynamics of a Finite Liquid Oxygen (LOX) Column in a Pulsed Magnetic Field

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I. Introduction

It is well known that liquid oxygen has a sufficient paramagnetic susceptibility that a strong magnetic field gradient can lift it in the earth's gravitational field.

The movement of liquid oxygen is vital to the space program since it one of the primary oxidizers used for propulsion. Transport of liquid oxygen (LOX) via direct interaction of the magnetic fields (B field) with the fluid is a current topic of research and development at Kennedy Space Center, FL. This method of transporting (i.e. pumping) LOX may have particular advantages on Mars and other reduced gravitational environments, namely safety and reliability.

This paper will address transport of a magnetic fluid, LOX, via phased-pulsed electromagnets acting on the edge of the column of fluid. The authors have developed a physical model from first-principles for the motion of a magnetic fluid in a particular U-tube geometry subjected to a pulsed magnetic field from an arbitrary solenoidal electromagnet. Experimental data that have been collected from the analogous geometry correlate well to that of the ab-initio calculations.

II. LOX Dynamics in an Applied Magnetic Field

The force density on a column of LOX is given by [2]:

$$\mathbf{F}_B = \frac{1}{\mu_0}\left(\frac{\chi}{1+\chi}\right)(\mathbf{B}\cdot\nabla)\mathbf{B} \tag{1}$$

where μ_o is the permeability of free space and χ is the volume magnetic susceptibility of the LOX. LOX is paramagnetic having Currie-Weiss behavior with a Weiss temperature of –40.6 K. The magnetic susceptibility of LOX is 0.0053 at 57 K and.0035 at 90 K. All of the results presented below were taken in a liquid nitrogen (LN2) bath with an assumed susceptibility for LOX of 0.0042.

The pulsed coils generate an insignificant electric field variation in time and hence the curl of the magnetic field is nearly zero. Using this result in equation 1 leads to a gradient form for the magnetic force density:

$$\mathbf{F}_B = \frac{1}{2\mu_0}\left(\frac{\chi}{1+\chi}\right)\nabla|\mathbf{B}|^2 . \tag{2}$$

The magnetic force field across a tube coincident with the coil axis was first calculated using exact analytic equations. As shown in Figure 1, this analysis demonstrated that the z-component of the magnetic force field is approximately constant across a cross section of the tube and nearly equal to the field at the center of the coil.

Figure1 shows that the standard expression for the magnetic field along the axis of a single wire loop as given in equation 3 is an adequate approximation for the magnetic field at all points within the LOX column

$$B_z = \frac{\mu_0 a^2 i(t)}{2(a^2 + z^2)^{3/2}} \quad . \tag{3}$$

The solenoid geometry used to calculated the magnetic fields is shown in Figure 2. Here a is the radius of the inner coil, b is the radius of the wire itself, dx is the wire spacing in the radial direction, dz is the wire spacing in the z direction, Mx is the number of layers in the solenoid, and Mz is the number wraps in the z direction.

The magnetic field at any location along the axis of the solenoid can now be written as the sum of magnetic fields contributions from each coil in the solenoid

$$B_z(t) = \frac{\mu_0 i(t)}{2} \sum_{m=-(M_z-1)/2}^{(M_z-1)/2} \sum_{n=0}^{M_x-1} \left(\frac{(a+nd_x)^2}{\left[(a+nd_x)^2 + (z-md_z)^2\right]^{3/2}} \right) \quad , \tag{4}$$

where $i(t)$ is the current passing through the solenoid.

Figure 3 shows a diagram of the LOX U-tube and solenoid. The LOX fills this tube to a height, *l0*, and has a base length given by *l1*, resulting in a total LOX column length, *L*, of *2l0+l1*. The solenoid is located at the liquid to gaseous interface so that the lower portion of the solenoid overlaps liquid while the upper portion overlaps gas. Upon applying current through the solenoid, the magnetic field/gradient results in an upward directed force on the LOX column that is maximal when the LOX edge is at the midpoint of the solenoid, and approaches zero when the LOX column is near the top of the solenoid.

A set of finite difference equations used to model the motion of the LOX column are given below in the order in which they were calculated. The area of the LOX column is given by, A, the time step count by the subscript k, the density of LOX by ρ, and the velocity of the column by v. Note that the volume integral of equation 2 using the magnetic field from equation 4 yields a total force term proportional to the difference in the magnetic field at the top of the LOX column and a point far from the coil. This is an intuitively useful result because it shows that the total magnetic force on the column is dependent on the magnetic field at only two regions in space, the top surface (equation 6) and the bottom surface (equation 7) of the column.

$$z_k = z_{k-1} + v_{k-1}\Delta t + \frac{F_{k-1}}{2(A\rho L)}\Delta t^2 \tag{5}$$

$$B_k = \frac{\mu_0 i(t)}{2} \sum_{m=-(M_z-1)/2}^{(M_z-1)/2} \sum_{n=0}^{M_x-1} \left(\frac{(a+nd_x)^2}{\left[(a+nd_x)^2 + (z_k-md_z)^2\right]^{3/2}} \right) \tag{6}$$

$$B_{0k} = \frac{\mu_0 i(t)}{2} \sum_{m=-(M_z-1)/2}^{(M_z-1)/2} \sum_{n=0}^{M_x-1} \left(\frac{(a+nd_x)^2}{\left[(a+nd_x)^2 + (z_k-L-md_z)^2\right]^{3/2}} \right) \tag{7}$$

$$F_k = \frac{A\chi}{2\mu_0(1+\chi)}\left(B_k^2 - B_{0k}^2\right) - \rho\, gAh - \mathrm{sgn}(v_k)(\beta|v_k| + \gamma|v_k|^2) \tag{8}$$

The total force on the LOX column, given by equation 8, is due to three forces: a magnetic force that is affected by the solenoid geometry; a gravitational force that is affected by U-tube geometry; and a damping force modeled empirically by a 2nd order polynomial with coefficients β and γ. Note that the force in equation 8 is in Newtons and not force densities as in equations 1 and 2, the latter having been integrated over the LOX volume. The remaining dynamic equations and definitions are given in equations 9 through 12.

$$v_k = v_{k-1} + \frac{F_k + F_{k-1}}{2(A\rho L)}\Delta t \tag{9}$$

$$h_k = \begin{cases} 2\Delta z_k & |\Delta z_k| < l_0 \\ \Delta z_k + \mathrm{sgn}(\Delta z_k)\ l_0 & l_0 \le |\Delta z_k| < L - l_0 \\ \mathrm{sgn}(\Delta z_k)\ L & |\Delta z_k| \ge L - l_0 \end{cases} \tag{10}$$

$$\Delta z_k \equiv z_k - z_0 \tag{11}$$

$$L \equiv 2l_0 + l_1 \tag{12}$$

Using Equations 5 through 12, the dynamic motion of a LOX column affected by a solenoid's magnetic field can be calculated numerically.

III Experimental Setup

Upon first consideration of equations 5 through 12 one might suggest using a high magnetic field intensity for a LOX pump due to the field squared term in equation 8. With this in mind several small solenoids were constructed (typical parameters, Mx=31, Mz=19, a=9 mm, 30 gage wire), which were driven by discharging a 470 uF cap charged to 400 V. At LN2 temperatures this resulted in over 100 Amps of current and a peak magnetic field of about 6 Tesla. The resultant upward pressure on the LOX column was 60,000 Pascals, or 0.6 atm, but for only 3-4 milliseconds. It was expected that a substantial movement of the LOX column would be seen (the total column length was about 33 cm), but this was not the case. Instead, the results were erratic; often nothing appeared to happen when the capacitor was discharged, but sometimes the top of the LOX column was torn off and thrown up along the inner walls of the flow line. It was determined that positioning of this small coil relative to the liquid/gaseous interface was critical and that the pressures being created were near the cavitation point of LOX. While cavitation might be an interesting topic in its own right, the goal of the present work is to pump LOX. Consequently, a decision was made to lower the magnetic field intensity but compensate for this by increasing the drive time of the coil. This was achieved by replacing the high voltage capacitor with a low voltage, high current power supply. The

coils were redesigned from lower gage wire making them much longer and heavier, but capable of handling the thermal dissipation associated with such a drive.

The goal of the experimental setup was to correspond as closely as possible to the setup used in the numerical model. The experiment consisted of a pulsed solenoid surrounding a U-tube containing liquid oxygen. The current and the location of the LOX column must be measured as functions of time. The experiments were performed using a solenoid wrapped with round formvar-insulated copper wire or various gauges. The solenoids were wrapped directly on ¼" FEP 890 Teflon tubing with an inner diameter of 4.75 mm. The tubing where LOX was to be generated was in a squared U shape with one leg of the U containing the section with the solenoid (as sketched in Figure 3).

Gaseous helium was used to purge the system of any air, ensuring that the only liquid produced would be liquid oxygen. After the system was completely purged with helium, the U-shaped tube was placed in a transparent liquid nitrogen filled Dewar. Gaseous oxygen was then introduced into the system with subsequent liquefaction of the gas in contact with the liquid nitrogen. The valve setup was such that, after purge, the pressure of gaseous oxygen on both sides of the LOX column in the U-tube would be equal

The solenoid coil was subjected to a current pulse through a high-power Insulated Gate Bipolar Transistor (IGBT). The IGBT was controlled with a 10 Volt square wave pulse generated by National Instruments 6035e MIO card. The current through the magnet was monitored by measuring the voltage across a small resistor in series with the solenoid using the same MIO card.

The liquid oxygen level was measured using a National Instruments Vision system 1400 which consists of a RS-170 camera and video capture board. The images acquired were at 30 Hz and precisely synchronized with the pulse of current through the solenoid and the measurement of the current. Due to the rapid motion of the LOX level edge and the slow rate/exposure of the camera, the edge was found manually and converted from pixels to real units using a calibrated grid in the field of view of the camera (see Figure 4).

IV Results and Discussion

Figure 5 shows a set of typical results for a coil composed of 1902 wraps (Mx = 27 and Mz about 71) wrapped with 18 gauge wire, resulting in a solenoid about 8.3 cm long with a resistance of about 0.5 Ohm at 77 K. Thirty amp current pulses of three different durations, 191 ms, 432 ms, and 1000 ms, were sent through the solenoid and the level of the LOX column measured. In each case the level of the LOX column was a short distance above the center of the solenoid when the current pulse was initiated. The solenoid location is shown on the right side of the plot. The total length of the LOX column was 36 cm. The resultant magnetic field of about 0.9 Tesla accelerated the LOX column upward past the top of the solenoid. For the 191 ms and 432 ms pulses, the current was turned off before the LOX column reached its peak height, removing the magnetically induced force. The LOX column then dropped back down and oscillated about its original rest position. The 1000 ms pulse was sufficiently long to allow the LOX column to oscillate once about a new equilibrium position located near the top of the solenoid.

Figures 6 and 7 show the motion of the LOX column versus starting height for two different coils when driven with pulses of one second duration. The coil used in Figure 6 was the same as that described above while the coil used in Figure 7 was composed of 1792 wraps of 21 gauge wire (Mx = 18 and Mz is about 100). In each case the primary effect of the magnetic field was to lift the LOX column to a new equilibrium position, about 3 cm high, about which it oscillated. After the current was turned off, the LOX column dropped and oscillated about its starting position. As before, the solid lines represent the model fits using using $\beta = 0.007$ and $\gamma = 0.11$, and the symbols represent the experimental data.

The dynamics of the LOX column are especially sensitive to starting position near the ends of the solenoid because the large gradient in the magnetic field. Variations in starting distance of as little as one millimeter can result in two or more centimeter variations in maximum displacement of the column.

If damping forces are neglected, equations 5 through 12 can be solved in closed form for the case of a constant current, yielding an oscillating column of fluid. Devoid of dampling forces, the period of oscillation is exactly that of a harmonics oscillator and found to be (recall that L is the length of the entire LOX column which varies from 30 to 40 cm in the above figures)

$$\tau = 2\pi\sqrt{\frac{2L}{g}}. \tag{13}$$

This predicted oscillation period is slightly smaller than that seen in the experimental data, which is expected since the effect of the damping forces would lengthen this cycle. However, as seen by the comparison between the experimental data and the numerically computed fit, the oscillation period is still longer than predicted. This may indicate a more complicated damping force dependence on velocity than assumed in the model.

V Conclusions

It has been demonstrated experimentally and modeled numerically that a solenoid, placed at the liquid to gas interface of a LOX column can be used to dynamically lift the column and even to propel it beyond the end of the solenoid. Under earth gravity the maximum distance a LOX column can be propelled is limited to a few inches, dependent upon the size of the column, the specifics of the solenoid and the available current and voltage. Howeverin reduced gravitational environments, such as on Mars or in Space, there may be a need to transport small amounts of liquid oxygen and in these lower gravitational fields, significant transport should be possible. Indeed, work is ongoing to produce LOX from the Martian atmosphere and from a reliability standpoint, it may be advantageous to use a pumping mechanism that requires no moving parts , as described in this paper, rather than a standard mechanical pump that would be more prone to failure. The requirements for these applications are evolving, but the above work has led to the development of a numerical model that can be used to develop such a device.

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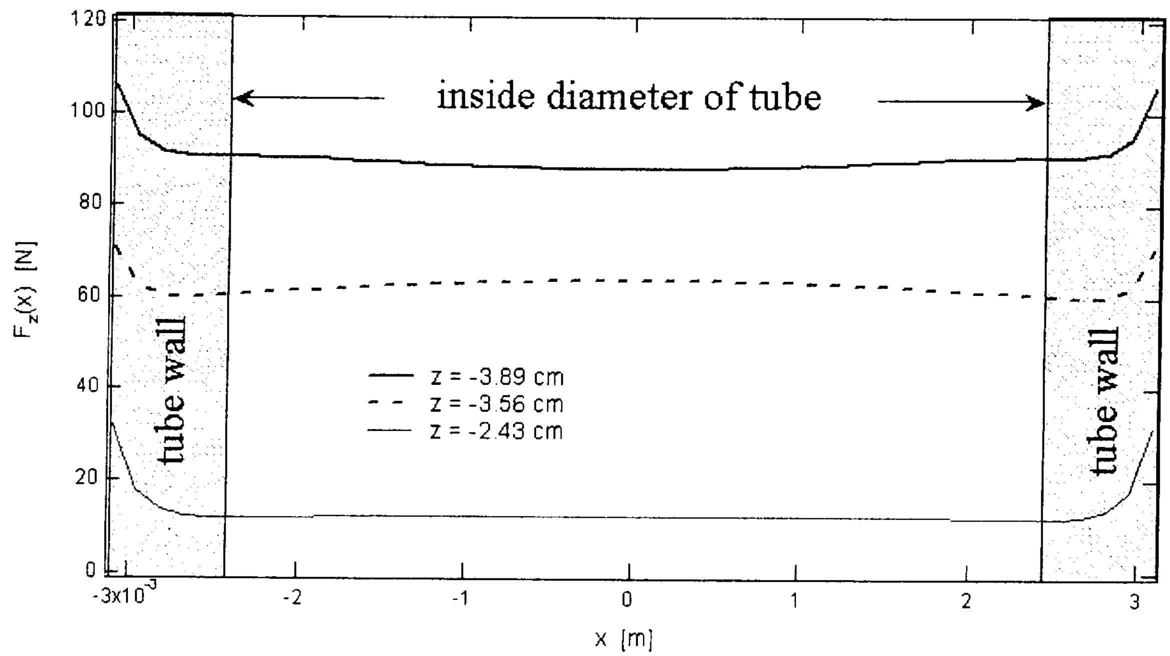

Figure 1. The magnetic force field calculated from exact analytic expressions for a given solenoid showing that the field is approximately uniform over the cross section of the tube and nearly equal to the field at the center.

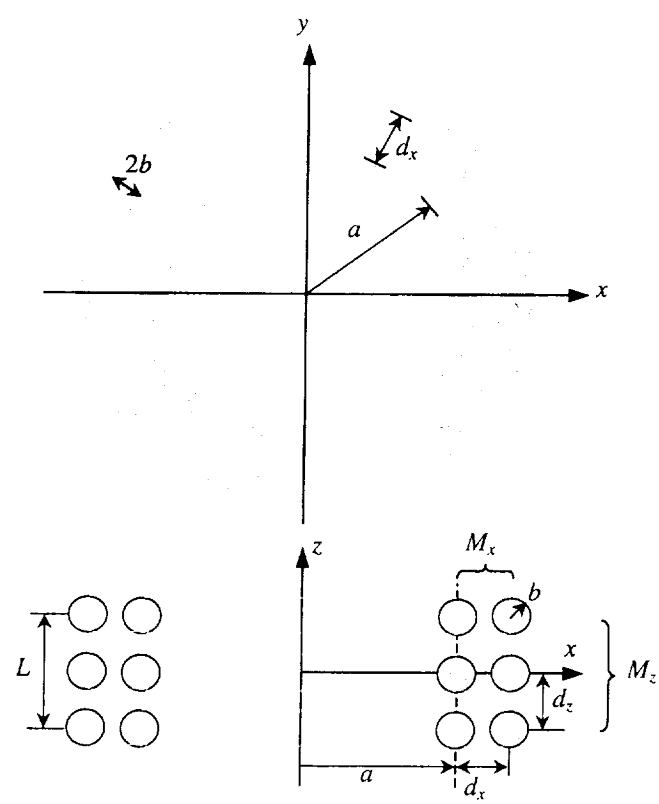

Figure 2. Stacked multiple layer coil.

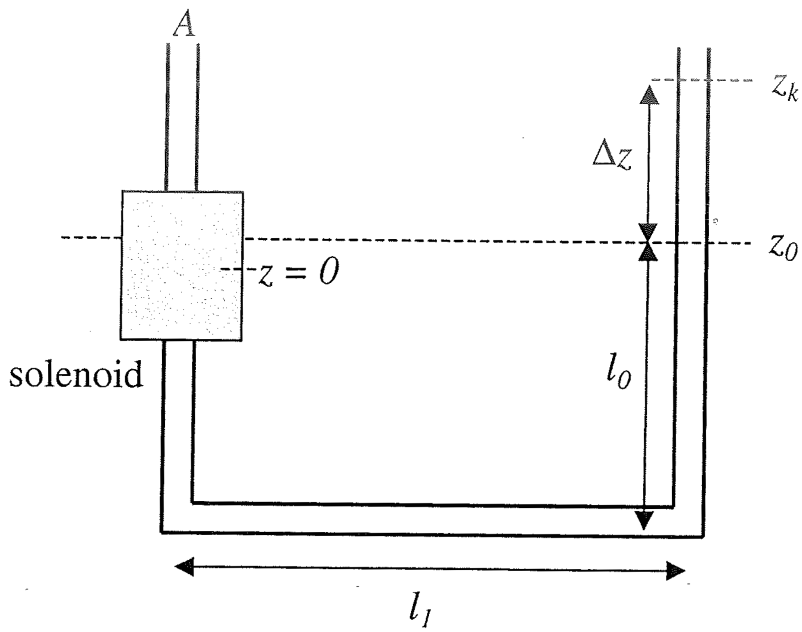

Figure 3. U-tube model

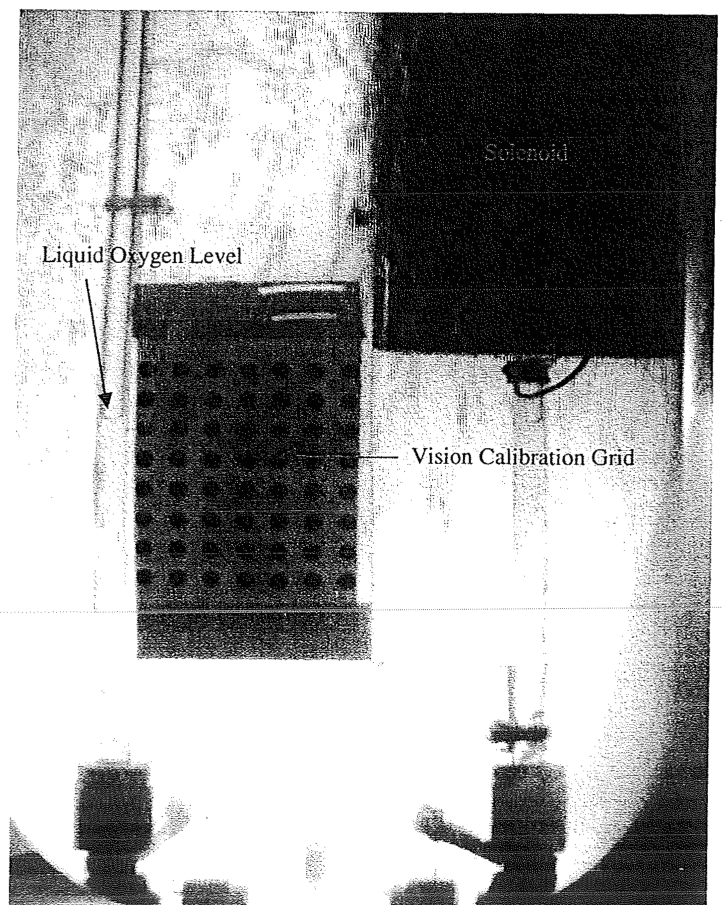

Figure 4 Experimental Setup.

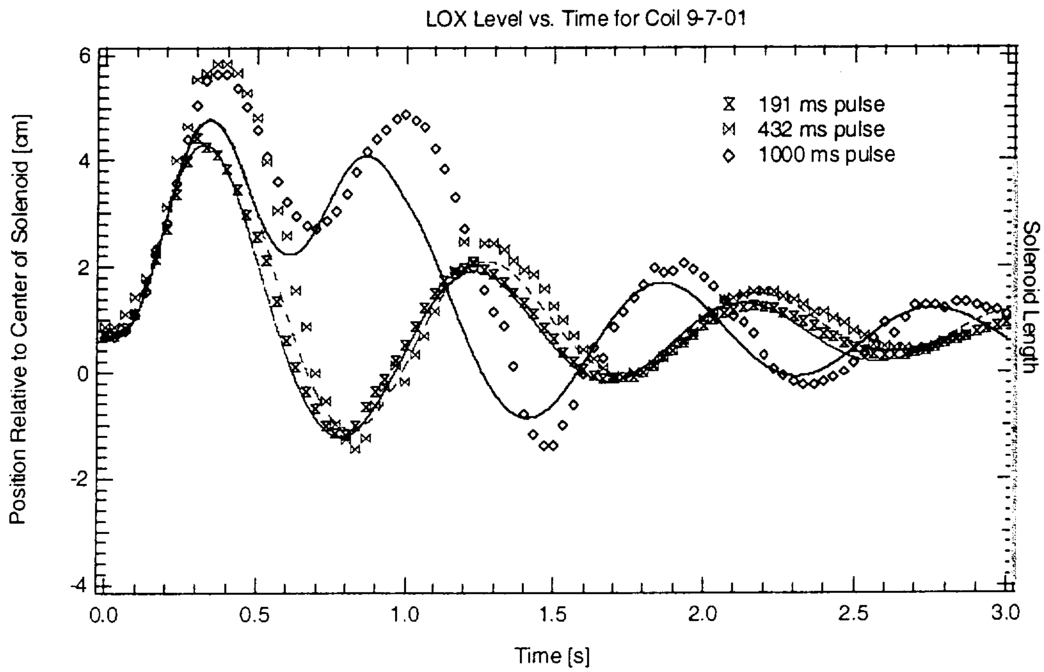

Figure 5. LOX column response to current pulses of different durations. The solid lines were generated by the iterative use of equations 5 through 12 and represent "best fits" to 21 datasets using $\beta = 0.007$ and $\gamma = 0.11$.

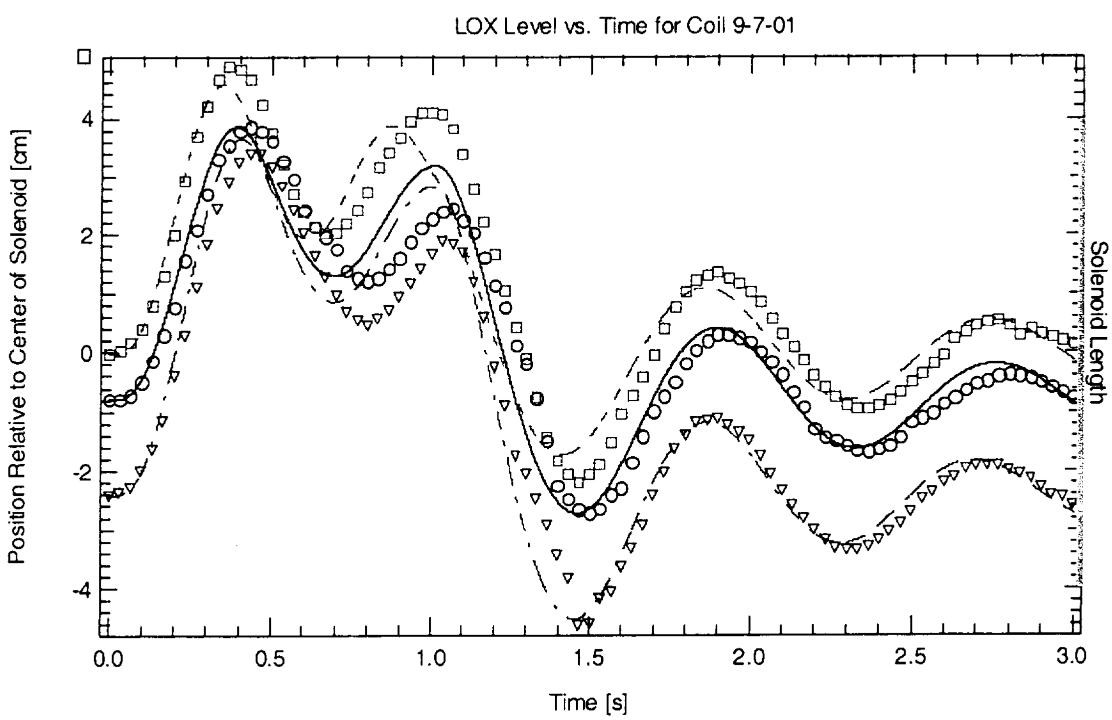

Figure 6. LOX column motion versus starting height.

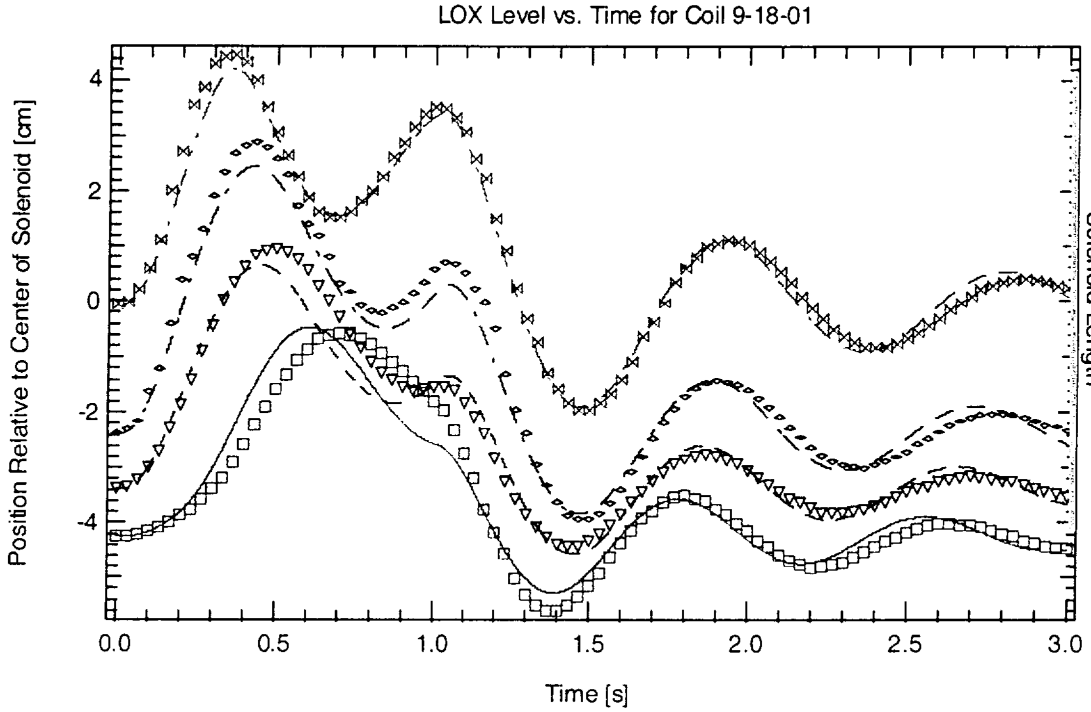

Figure 7. LOX column motion versus starting height